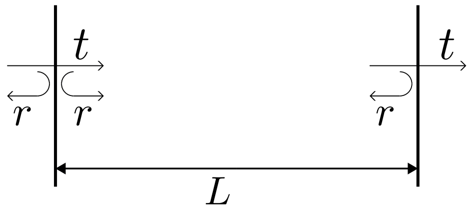
Problem Set 9 - LV 141.246 QISS - 18.6.2012

1. Resonance



As demonstrated in class, one gets a resonance with two scatterers

$$\begin{aligned} S &= te^{ikL}t \\ &\quad + te^{ikL}re^{ikL}re^{ikL}t \\ &\quad + ... \\ &= \frac{t^2 e^{ikL}}{1 - r^2 e^{i2kL}} \end{aligned}$$

Typically the transmission coefficient is very small, $t \ll 1$. Furthermore the total probability has to be preserved $r^2 + t^2 = 1$. Note that $k = \omega/c$, where c is the propagation speed of the wave.

(a) Identify the resonances. Show that the expression in the vicinity of the resonance has the form

$$S_L = \frac{i\gamma}{\omega - \omega_0 + i\gamma}$$

What is γ?

(b) What is the transmission on resonance, width of the resonance (FWHM-full width half max) $\delta\omega$ and the quality factor $Q = \omega_0/\delta\omega$.

2. Jaynes-Cummings Hamiltonian

$$\mathcal{H} = \hbar\omega_r a^\dagger a - \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma_- + a\sigma_+)$$

(a) Write the Jaynes-Cummings Hamiltonian in the basis $|g,0\rangle$, $|e,0\rangle$, $|g,1\rangle$, $|e,1\rangle$, $|g,2\rangle$, ...

(b) Solve the n excitations manifold and show that the level difference on resonance $\omega_r = \omega_q$ scales with $\sqrt{n}$.

(c) Solve the problem for the dispersive regime, i.e. $\Delta = \omega_r - \omega_q \gg g$ (First order g/Δ).